Exercise 10

For the following exercises, use long division to divide. Specify the quotient and the remainder.

$$(x^3 - 126) \div (x - 5)$$

Solution

Set up the division problem, writing out every term in the dividend.

$$x - 5 \overline{)x^3 + 0\,x^2 + 0\,x - 126}$$

Divide the leading term of the dividend by the leading term of the divisor and place the result above the term with the same power of x.

$$
\begin{array}{r}
x^2 \\
x - 5 \overline{)x^3 + 0\,x^2 + 0\,x - 126}
\end{array}
$$

Multiply this result by the divisor and subtract it from the dividend.

$$
\begin{array}{r}
x^2 \\
x - 5 \overline{)x^3 + 0\,x^2 + 0\,x - 126} \\
\underline{-\left(x^3 - 5\,x^2\right)} \\
5\,x^2
\end{array}
$$

Bring the next term in the dividend down.

$$
\begin{array}{r}
x^2 \\
x-5\overline{\smash{\big)}\,x^3+0\,x^2+0\,x-126} \\
-\underline{\left(x^3-5\,x^2\right)}\Big\downarrow \\
5\,x^2+0\,x
\end{array}
$$

Divide the leading term of this modified dividend by the leading term of the divisor and place the result above the term with the same power of x.

$$
\begin{array}{r}
x^2+5\,x \\
x-5\overline{\smash{\big)}\,x^3+0\,x^2+0\,x-126} \\
-\underline{\left(x^3-5\,x^2\right)} \\
5\,x^2+0\,x
\end{array}
$$

Multiply this result by the divisor and subtract it from the modified dividend.

$$
\begin{array}{r}
x^2+5\,x \\
x-5\overline{\smash{\big)}\,x^3+0\,x^2+0\,x-126} \\
-\underline{\left(x^3-5\,x^2\right)} \\
5\,x^2+0\,x \\
-\underline{\left(5\,x^2-25\,x\right)} \\
25\,x
\end{array}
$$

Bring the next term in the dividend down.

$$
\begin{array}{r}
x^2 + 5x \\
x - 5 \overline{)\,x^3 + 0x^2 + 0x - 126} \\
-(x^3 - 5x^2) \\
\hline
5x^2 + 0x \\
-(5x^2 - 25x) \\
\hline
25x - 126
\end{array}
$$

Divide the leading term of this modified dividend by the leading term of the divisor and place the result above the term with the same power of x.

$$
\begin{array}{r}
x^2 + 5x + 25 \\
x - 5 \overline{)\,x^3 + 0x^2 + 0x - 126} \\
-(x^3 - 5x^2) \\
\hline
5x^2 + 0x \\
-(5x^2 - 25x) \\
\hline
25x - 126
\end{array}
$$

Multiply this result by the divisor and subtract it from the modified dividend.

$$
\begin{array}{r}
x^2 + 5x + 25 \\
x - 5 \overline{)\,x^3 + 0x^2 + 0x - 126} \\
-(x^3 - 5x^2) \\
\hline
5x^2 + 0x \\
-(5x^2 - 25x) \\
\hline
25x - 126 \\
-(25x - 125) \\
\hline
-1
\end{array}
$$

There are no further terms in the dividend to drop down, so the division is complete. The quotient is $x^2 + 5x + 25$, and the remainder is -1.

$$
(x^3 - 126) \div (x - 5) = x^2 + 5x + 25 + \frac{-1}{x - 5}
$$